The Key to Newton's Dynamics

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The Kepler Problem and the *Principia*

Containing an English Translation of Sections 1, 2, and 3 of Book One from the First (1687) Edition of Newton's *Mathematical Principles of Natural Philosophy*

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with English translations from the Latin by
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CONTENTS

PREFACE

Early in the seventeenth century, the astronomer/mathematician Johannes Kepler demonstrated that the movement of the planet Mars is best described as elliptical motion about the sun located at a focus of the ellipse. Late in the seventeenth century, the challenge still remained for astronomers to determine the nature of the force required to maintain elliptical motion about a focal force center: the Kepler problem. The key to Newton's dynamics resides in his solution of the Kepler problem. It is the goal of this book to make a detailed explanation of that solution available to a wide range of interested students and scholars. Newton's answer provides the analytical basis for the concept of a universal gravitational force. Much has been written on the *ramifications* of this solution, but the details of the solution are rarely made available to any but the expert in the field. The historian of science may be deterred by the mathematical details, the scientist by the conceptual details, and the student by both sets of details. When these details are provided, however, there appears a surprisingly clear and simple analytical structure that frames Newton's speculation concerning the role and nature of force. This structure arises in Newton's early work at Cambridge (pre-1669); it continues to develop after his revival of interest in the problem after 1679; and it achieves its fruition in the first three sections of the first edition of his *Mathematical Principles of Natural Philosophy* (*Principia*) in 1687.

Chapters 1 and 2 of this book set the Kepler problem in historical and conceptual perspective with all reference to mathematical detail postponed. The object is to set forth clearly the challenge of the direct problem of elliptical planetary motion and to supply the conceptual tools employed in its solution, in particular Newton's debt to the works of both

Descartes and Galileo. Chapter 3 presents a detailed discussion of two of Newton's early (pre-1669) analyses of uniform circular motion. In chapters 4 to 6, Newton's solution to the specific direct problem of elliptical planetary motion is examined in detail as it appears in the set of four theorems and four problems that he sent to Halley in 1684 preliminary to the publication of the *Principia.* Chapters 7 to 9 explore the revisions and extensions that are made to these basic elements in the first and revised editions of the *Principia,* and chapter 10 transforms the basic theorems into modern mathematical dress. The book concludes with a translation into English of the first three sections of Book One of the first edition (1687) of the *Principia.* The first edition has rarely been translated and its choice here provides a capstone for the detailed analyses of this book. It also provides a comparison to the existing translations of the third edition (1726) in a direct fashion not available in a variorum edition.

I have used portions of this book in an undergraduate course in the history of science; the students needed only a general high-school background of basic mathematics and science. Specifically, I have used the theorems and problems from the tract *On the Motion of Bodies in Orbit* (*On Motion*) from chapters 4 and 5. In one class period I assigned the details of Theorem 1 (the area law) and the details of Theorem 3 (the force law). In a second class period I assigned the details of Problem 1 (a simple application) and then discussed the solution of Problem 3 (elliptical planetary motion). Other sections were assigned as supplementary reference material. I have also used the entire book as a text for an advanced junior/senior undergraduate course in the history of science, usually as a tutorial. It should serve the same function as a graduate text in departments of history of science.

This book is intended, however, for scholars as well as for students. My choice to study the details of a single problem, however important, may seem overly restrictive. Scholars are interested in the development and growth of Newton's thought on the nature and source of gravitational force, and his reflections on the very nature of scientific analysis itself. A continuity of method, however, lies beneath the changing vocabulary and developing techniques of Newton's work. His method is revealed only by a study of the *details* of the solutions in his early and later work. While that method itself may not be sufficient to reveal Newton's innermost thoughts, nevertheless it provides a measure against which speculations can be held. Consider, for example, the claim often made that Newton's early work reveals a confusion concerning force that was later eliminated, specifically that he attempted to combine two or more different force concepts. It is my contention that an analysis of the details of Newton's solution reveals no such confusion. One must understand the details in order to make an no such confusion. One must understand the details in order to make an informed decision. As a second example, consider the question of the debt owed by Newton to Robert Hooke on the nature of celestial dynamics. The debate revolves about Newton's switch in terminology from *centrifugal* to *centripetal* force following his correspondence with Hooke in 1679. I argue that a close inspection of the details of the post-1679 solution reveals that Newton's method did not change from the method used before 1669. The method survives even if there is a conceptual shift. The truth of that claim lies buried in the trivia of the solution.

I was encouraged to produce such detailed analyses of Newton's solution by my late dear friend and close colleague, Professor Betty Jo Teeter Dobbs. Her death brought a great loss to the world of Newtonian scholarship and to all who knew her. She will be greatly missed. In a letter I received from her after she read the opening sections of my manuscript, she asked if the essentials of the solution could not be presented, as she put it, "without all of that $QR / QT^2 \times SP^2$," by which she meant the analytical details. My reply to her, as it is to all, is that it is not possible and that the task of reading them is really not that formidable. Moreover, the result is worth the effort. Newton's solution to the direct problem of elliptical motion does indeed supply the key to the dynamics that provided the basis for the concept of universal gravitational force.

JB^2

ACKNOWLEDGMENTS

A major portion of this book was written with the support of a grant from the Humanities, Science, and Technology division of the National Endowment for the Humanities (NEH). It is their goal to encourage new translations of and guided studies to major works of science that are of interest to both humanists and scientists. The work of Isaac Newton is widely acclaimed as the most influential in Western science and as such was a logical choice for their support. The project began in January 1990 and the final manuscript was delivered to the publisher in September 1994. During this time, I received help and encouragement from several sources.

In addition to the NEH, I would like to acknowledge the kindness shown by the staff of the library of the Royal Society. They provided a home away from home during my time in London. I would also like to express my gratitude to Lawrence University for a sabbatical leave granted during the project, as well as the summer research support they gave for a student colleague in 1991. Moreover, I had full use of the computer facilities of the university for preparing both the manuscript and the drawings. I am indebted to my colleagues in the department of physics, David Cook and John Brandenberger, for their support and to Bruce Pourciau of the department of mathematics for his patient explanations. In a larger sense, the liberal arts tradition of the university provided the major encouragement. The interdisciplinary nature of the freshman studies program at Lawrence University provided the initial motivation for a physicist to begin to explore with undergraduates the work of Plato, Aristotle, Galileo, and finally Newton. The administration encouraged me to offer a course in the history of science, and my colleagues in the humanities and sciences supported the effort.

It will be clear that this book owes a great debt to Tom Whiteside, the editor of the eight volumes of Newton's mathematical papers. This remarkable publication, a labor of twenty-two years, comprises a collection of Newton's original mathematical papers; Whiteside's translation of them from the Latin provides an invaluable resource for anyone interested in Newton's dynamics or mathematics. Moreover, Whiteside has supplied extensive notes and commentaries that offer the reader historical and mathematical insights. In addition, he has been most generous with his time in correspondence and conversation. I am also grateful to I. Bernard Cohen, who has produced a new English translation of the entire text of the third edition of Newton's *Principia*. I had the privilege of reading portions of an early draft of his translation and benefited greatly from it. He has been a constant source of encouragement. My thanks go to colleagues who have read sections of the first few chapters and provided commentaries—Jo Dobbs, Herman Erlichson, Ivor Grattan-Guiness, and Peter Spargo—and to Michael Nauenberg for his comments on the final chapter. Although they have not seen the entire manuscript and are not responsible for the opinions expressed in it, I have profited from their observations. I also wish to thank Alan Shapiro for the extremely useful commentary he provided as a referee for an earlier version of the manuscript.

This work also drew heavily upon my student colleagues at Lawrence University. I first read the opening sections of the *Principia* with two physics majors, Bob Hanisch and Gene Peterson, when we were at the Lawrence University London Center. Paul Stieg did a senior seminar in which we explored in detail the first three sections of Book One. Outstanding were the contributions of Andrea Murschel, who spent the summer of 1991 as my research assistant at Lawrence University and continued in her senior year to bring her extensive linguistic and analytic skills to bear on the project. In 1992 she was awarded a National Science Foundation fellowship in the history of science and she shows great promise of becoming an outstanding scholar. Credit is also due many of my other students at Lawrence University. In class, they worked through the sections of the tract *On Motion* to be found in chapters 4, 5, and 6, and they served as a check on the accuracy and clarity of my explanations. Their only reward, beyond the joy of learning, was induction at the end of the term into membership of the Loyal Society of S.I.N. (Sir Isaac Newton) with its toast, made with cider of course: "Up with gravity, down with levity."

Finally, and in the place of honor, I want to thank my wife, Dr. Mary Ann Rossi, the classics scholar who translated into English the tract *On Motion* and the selections from the first edition of the *Principia*. Long hours were spent as I, the physicist, argued for what I thought was Newton's intent, and she, the Latinist, argued for what Newton's Latin actually said. In addition, she took time from her other scholarly activities to read and comment upon the various drafts of my manuscript. One is fortunate to have an agreeable companion in one's personal or professional life; I am doubly fortunate to have found one who serves that role in both spheres. This project has been rewarding, but it pales in comparison to our earlier joint venture that produced daughters Lynn and Sandy and sons Rob and Scot. This work is most affectionately dedicated to the memory of our daughter Sandy (12 October 1958–3 February 1995).

PART ONE

The Background to Newton's Solution

Bust of Isaac Newton, by L. F. Roubiliac (c. 1737), currently located in the entrance hall of the Royal Society in London. Newton was elected to the Royal Society in 1662 and served as its president from 1703 until his death in 1727. Copyright © The Royal Society. Reproduced by permission.

ONE

A Simplified Solution

The Area Law, the Linear Dynamics Ratio, and the Law of Gravitation

Isaac Newton's *Philosophiae Naturalis Principia Mathematica* (The mathematical principles of natural philosophy), hereafter referred to as the *Principia*, justifiably occupies a position as one of the most influential works in Western culture, but it is a work more revered than read. Three truths concerning the *Principia* are held to be self-evident: it is the most instrumental, the most difficult, and the least read work in Western science. A young student who passed Newton on the streets of Cambridge is reported to have said, "There goes the man who writ the book that nobody can read." It fits Mark Twain's definition of a classic as a work that everyone wants to have read but that nobody wants to read. The essential core of the *Principia*, however, does not lie beyond the reach of any interested and open-minded individual who is willing to make a reasonable effort.

In 1693, Richard Bentley, a young cleric who was later to become Master of Newton's college, wrote to ask Newton for advice on how to master the work. Newton suggested a short list of background materials, and then, concerning the *Principia* itself, advised Bentley to read only the first three sections in Book One (i.e., the first sixty pages of the four hundred pages that make up the first edition). These sections provide the theoretical background for the astronomical applications that Newton presented in Book Three and regarded as of popular scientific interest. In the introduction to Book Three, Newton repeated the advice that he had given to Bentley:

> I had composed the third book in a popular method so that it might be read by many. But since those who had not sufficiently entered into the principles could not easily discern the strength of the consequences nor put aside long-held prejudices, I chose to rework the substance of that book into the form of propositions in the mathematical way, so that they might be read only by

> those who had first mastered the principles. Nevertheless, I do not want to suggest that anyone should read all of these propositions—which appear there in great number—since they could present too great an obstacle even for readers skilled in mathematics. It would be sufficient for someone to read carefully the definitions, laws of motion, and the first three sections of the first book; then let [the reader] skip to this [third] book.[1]

Newton's sage advice to the general reader to concentrate on the first three sections of Book One of the *Principia* appeared in the first edition of 1687 and remained unchanged in the two revised editions published in 1713 and 1726, all during Newton's lifetime. It is the third and final edition that has been reproduced in many subsequent editions and translated into many other languages. Because this third edition is readily available and because it is seen to represent Newton's most fully developed views, it is almost exclusively taken as a basis for the study of Newton's dynamics. The general reader, however, should not begin with this final edition and its many additions and revisions, but rather with the first edition and its relatively straightforward presentation.

In 1684, Newton sent to London a tract entitled *On the Motion of Bodies in Orbit (On Motion)* that was to serve as the foundation for the first edition of the *Principia* of 1687. This comparatively short tract presents in a clean and uncluttered fashion the basic core of Newton's dynamics and its application to the central problem of elliptical motion. The brief set of definitions that appeared in *On Motion* was expanded in the *Principia* into a much larger set of definitions, laws, and corollaries. Further, the first four theorems and four problems in *On Motion* were expanded into fourteen lemmas and seventeen propositions in the *Principia.* (Theorem 1 of *On Motion* is Proposition 1 of the *Principia* but Problem 4 of *On Motion* is Proposition 17 of the *Principia*). The expanded framework of numbered propositions by itself, however, does not tell the entire story. Even more troublesome for the general reader is Newton's practice of adding new material to the old framework. Having established the expanded set of propositions and lemmas in the early draft of the first edition, Newton elected to hold to that framework as he inserted additional material in his published revised editions. Even in the preface to the first edition, Newton apologized to his readers for such insertions.

> Some things found out after the rest, I chose to insert in places less suitable, rather than to change the number of the propositions as well as the citations. I heartily beg that what I have done here may be read with patience.[2]

After the publication of the first edition, Newton began work on a grand radical revision of the *Principia* in which many of the propositions would have been renumbered and retitled. In contrast to the single method of the first edition, Newton clearly presented three alternate methods of dynamic analysis in this projected revised scheme, each method set forth in a new proposition. Unfortunately, Newton never implemented this new scheme of renumbering the propositions and lemmas in the published revisions. If the challenge of renumbering the propositions and correcting the cross-references was too much in the limited first edition, then it was apparently overwhelming in the expanded revised editions. The new material added to the published revised editions simply was inserted into the old structure of the first edition. The third method of dynamic analysis, so clearly differentiated in the projected revision, was distributed throughout the theorems and problem solutions of the second and third sections of the published revisions. The reader of *On Motion* and, to a lesser extent, of the first edition is not faced with this difficulty. In those works, Newton clearly explicates his analysis with a single method applied uniformly to several problems; until the reader understands his original method and his unpublished restructuring, however, Newton's additions to the much studied revised third edition appear as distractions rather than enrichments.

A SIMPLIFIED SOLUTION

The story of Isaac Newton and the apple is a familiar one. We have all seen the portrayal of an English gentleman who is sitting under a tree and is struck on the head by a falling apple. In a flash, he leaps to his feet and runs off shouting about the theory of universal gravitation. The story has its foundation in Newton's own telling and is attested by a number of memoranda written by those close to him in his later years. The setting is the garden of his country home, the time is 1666, and Newton, a young man of twenty-four, is home after a few years at university. The apple tree that provides his inspiration stands in his front garden, and the fruit it bears is a yellow-green cooking apple called the Flower of Kent. One version of the story, told by Newton in his later years and recorded by an associate, John Conduitt, includes the following statement:

> Whilst he was musing in a garden it came into his thought that the power of gravity (which brought an apple from the tree to the ground) was not limited to a certain distance from the earth but that this power must extend much farther than was usually thought. Why not as high as the moon said he to himself and if so that must influence her motion and perhaps retain her in her orbit, where upon he fell to calculating what would be the effect of that supposition.[3]

There is evidence that Newton made a calculation comparing the moon's centrifugal force, a celestial event, with the local force of gravity, a terrestrial event. Since it is a calculation that could have been inspired by any falling object, why not an apple? That early calculation of 1666 did not supply the mathematical basis for the general demonstration that the force necessary to maintain a planet in an elliptical orbit about the sun located at a focus of the ellipse is inversely proportional to the square of the distance between the sun and the planet (i.e., the law of universal gravitation). It was late in 1684, after Edmund Halley's famous visit to Newton's rooms at Cambridge University, before Newton gave anyone a copy of such a proof—a proof which Newton claimed to have produced in 1679. The inspiration of the falling apple of 1666 required more than a decade to reach its final goal.

In 1684, Newton sent Halley a solution to the problem of planetary motion in the tract *On Motion.* That solution is expressed neither in the mathematics of classical geometry nor in the mathematics of contemporary differential and integral calculus. As such it is a challenge to the modern physicist as well as to the classical scholar. The outline of the solution, however, is not complicated; it is the details that provide the challenge. Newton adapted the linear kinematics of Galileo to the inertial dynamics of Descartes and determined the nature of the force necessary to maintain planetary motion as described by Kepler. If a constant linear acceleration A is acting on a body of mass m, then its displacement D is proportional to the constant acceleration A and the square of the time t (i.e., $D = (1/2)At^2$). If one adds to this simple kinematic relationship the dynamic relationship that the acceleration A is proportional to the force F (i.e., $F = mA$), then the force F is directly proportional to the displacement D and inversely proportional to the square of the time t (i.e., $F = (2m)\ D/\ t^2$). Newton's genius manifests itself in adapting this simple proportional relationship of constant rectilinear force, distance, and time to the more complex problem of the nature of the planetary force, which is not constant. Newton's unique contribution was the assumption that the variable force could be considered to be approximately constant over a very short period of time. The three elements that Newton generated to produce the solution can be set forth quite simply: first, the relationship that expresses the time in terms of the area; second, the relationship that expresses the force in terms of the displacement and the time (and hence in terms of the area); and finally, the relationship that expresses the force necessary for planetary motion in terms of distance (i.e., the demonstration that the gravitational force is inversely proportional to the square of the distance).

Theorem 1

The first element is the law of equal areas in equal times, demonstrated in figure 1.1. If the force acting on a body is always directed to a fixed point S, then the time required to travel from point P to point Q is proportional to the shaded area SPQ. If successive areas are generated in equal

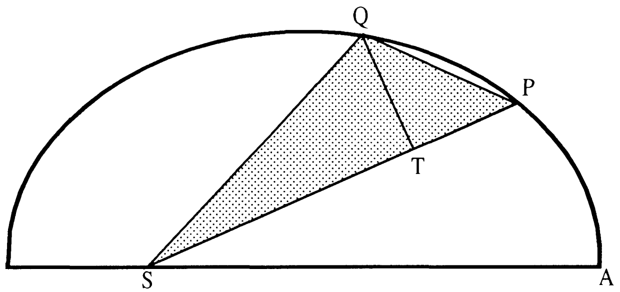

Figure 1.1 If a body moves from point P to point Q under a centripetal force directed toward the fixed point S, then the shaded area SPQ is proportional to the time.

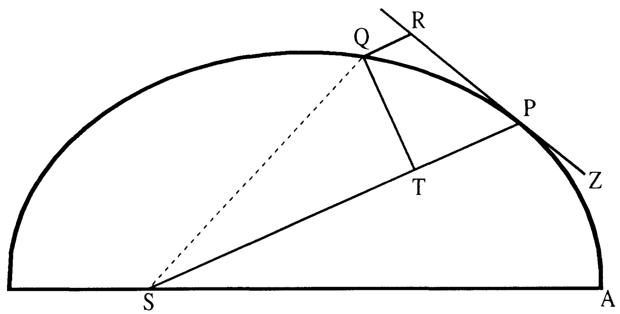

Figure 1.2 The force F required to maintain any orbit APQ about a center of force S is proportional to the displacement QR and inversely proportional to the area SQP.

times, then the areas swept out by the line from the body to the center of force are equal. This relationship was first recognized by the astronomer Johannes Kepler in 1609, but it was not until after 1679 that Newton demonstrated its general application to any motion under any force directed toward a fixed center. The area law is the link that was missing in Newton's earlier analysis of motion and it is the key element in his celestial dynamics; it appears as Theorem 1 in the 1684 tract *On Motion* and as Proposition 1 in the 1687 *Principia* (see chapter 4 for details).

Theorem 3

The second element is the basic relationship that I have elected to call the "linear dynamics ratio." Figure 1.2 is similar to Newton's diagram for Theorem 3 in *On Motion* and for Proposition 6 in the *Principia.* The line RPZ is the tangent to the curve APQ. If no force acted on a moving body at point P, then the body would continue in a given time interval along the tangent line to the point R. Because a force does act continuously on the body, however, it moves instead to the point Q. The displacement QR represents the deviation of the body from the tangential path PR due to the action of the force. Galileo had demonstrated in his experiments with inclined planes that for motion under a given constant force, the distance traveled is proportional to the square of the time. Newton assumes that as the point Q shrinks back to the point P, then the force can be treated as if it were constant. Thus, the distance QR is proportional to the square of the time and to the magnitude of the force at point P, or what is equivalent, the force is directly proportional to the distance QR and inversely proportional to the square of the time. From Theorem 1, the time is proportional to the triangular area SPQ and thus can be expressed in terms of the altitude QT and the base SP. The result is that the force F at point P can be expressed as follows:

$$\text{Force} \propto \text{distance} / (\text{time squared}) \propto QR / (QT^2 \times SP^2)$$

The challenge is to express the ratio QR/QT^2 in terms of SP and constants of the orbital figure, and hence to express the linear dynamics ratio $QR/(QT^2 \times SP^2)$, and thus the force, in terms of the radial distance SP (see chapter 2 for a review and chapter 4 for a detailed discussion of this theorem).

Problem 3

The third element is a demonstration by Newton of a relationship between portions of an ellipse. Figure 1.3 is a drawing of a planetary ellipse APQ with a focus at point S. The line LSL drawn through the focus S and perpendicular to the major diameter of the ellipse is called the *latus rectum L*. Newton demonstrates in Problem 3 and in Proposition 11 that as the point Q shrinks back to the point P, the ratio QR/QT^2 becomes equal to the reciprocal of the *latus rectum L*, which is a constant for a given ellipse. Thus, the force can be obtained quite simply from the linear dynamics ratio above:

$$\text{Force} \propto QR / (QT^2 \times SP^2) = 1 / (L \times SP^2) \propto 1 / SP^2$$

This result states that the force required to maintain a planet in an elliptical orbit about the sun located at a focus of the ellipse is proportional to the inverse square of the distance between the planet P and the sun S. Thus is demonstrated the mathematical basis for the law of universal gravitation, the essence of celestial interactions, which Newton provides for future astronomers and physicists (see chapter 2 for a review and chapter 5 for a detailed discussion of this problem).

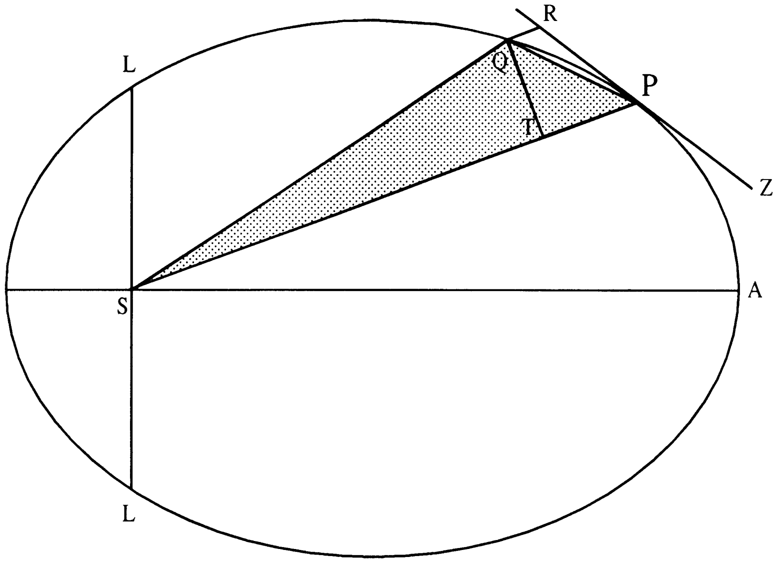

Figure 1.3 As point Q shrinks back to point P, the ratio QT^2/QR becomes equal to the line LSL, which is a constant (the *latus rectum*) for a given ellipse.

The details of the demonstrations of the relationships above are more demanding than is evident in this verbal gloss. Taken step by step, however, the analysis will become clear to the reader. At times Newton makes analytical leaps that for him are obvious and it is then my duty to supply the intervening steps. Thus, it is the number of steps rather than the size of any single step that offers the challenge. The reward for the patient reader is an insight into the solution of the problem of planetary motion, a problem that challenged astronomers for millennia. That solution is now universally held to have provided a major turning point in astronomy and natural philosophy in the late seventeenth century.

THE RECEPTION

Professional scholars, however, did not greet the publication of the 1687 *Principia* with unreserved praise. The dominant figure in seventeenth-century natural philosophy was the French scholar René Descartes, whose mechanical description of planets carried in a swirling vortex of celestial ether provided the model for many other natural philosophers. Two other outstanding figures in European mathematics and natural philosophy at the time of the publication of the first edition of the *Principia* were the Dutch scholar Christiaan Huygens and the German scholar Wilhelm Gottfried Leibniz. Both felt that Newton's description of the mathematical nature of gravitational force had failed to address the fundamental question of the physical cause of the force. It would appear that Huygens accepted the inverse-square law as a genuine discovery, although he believed that its cause remained to be investigated. Leibniz initially praised the 1687 *Principia* as one of the most important works of its kind since Descartes. He criticized Newton, however, for his rejection of Cartesian vortices and for his failure to provide an alternate physical cause for the gravitational attraction. In England, the astronomer and mathematician Edmund Halley served as the editor of the 1687 *Principia* and it was published under the imprimatur of the Royal Society. Even with this auspicious beginning, it was not without controversy that the *Principia* was finally published. The English scientist Robert Hooke claimed priority for the discovery of the inverse square nature of the gravitational force, a claim that Newton vehemently rejected. Despite individual reservations, the overall reception by the scholarly community was positive, and Newton established himself as one of the leading mathematicians of Britain and Europe. As one modern scholar of Leibniz's work put it, "Already in . . . 1695, Leibniz had abandoned the project of presenting a theory capable of competing with Newton's. Despite his subtle philosophical and theological objections, in the eighteenth century Leibniz had left Newton master of celestial mechanics."[4]

As the scholarly reputation of the *Principia* grew, even those who professed little or no mathematical ability came to pay homage. The English philosopher John Locke, in exile in Holland at the time of the book's publication, obtained assurance from Huygens that the mathematical propositions of the *Principia* were valid and then applied himself to understanding Newton's conclusions. Locke eventually referred to "the incomparable Mr. Newton" in the preface to his *An Essay Concerning Human Understanding*. The French writer and philosopher François Voltaire waxed even more eloquent when he drew the following comparison between Newton and the German astronomer Johannes Kepler: "Before Kepler, all men were blind. Kepler had one eye, Newton had two." The English poet Alexander Pope's often quoted heroic couplet, published shortly after Newton's death, revealed even more forcefully the popular view that Newton and the *Principia* opened doors that had long been closed: "Nature, and Nature's Laws lay hid in night: / God said, Let Newton be! and all was light." In the dedicatory poem to the first edition of the *Principia*, Edmund Halley reflected on Newton's "unlocking the treasury of hidden truth" and concluded that "nearer to the Gods no mortal may approach."

As the eighteenth century drew to a close, not everyone continued to praise the new world that appeared in Newton's work. Figure 1.4 is an early nineteenth-century caricature of Newton by the philosopher-poet-artist William Blake, who, putting imagination above reason, reacted negatively to the eighteenth-century veneration of Newton. In the portrait,

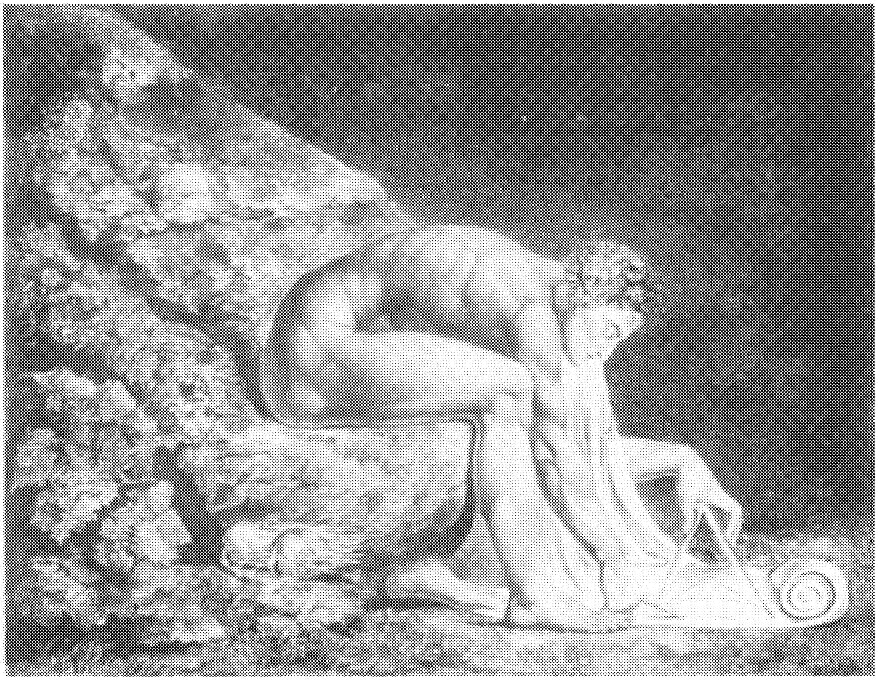

Figure 1.4 William Blake's portrait of Isaac Newton. Courtesy Tate Gallery, London.

triangles abound as the symbol of the geometrical and mathematical mentality that Blake opposed. Newton holds a triangular compass as he draws a triangular figure on the parchment, his fingers make triangles with the object in his hand, his legs form triangles with each other and with the rock on which he sits, the muscles of his body take on geometric forms that defy anatomical description, and triangular eyes scheme as they look down a triangular nose at geometric plans that triangular hands create below. For Blake, Newton symbolized the eighteenth-century regard for human reason that placed God above and separate from women and men, while Blake regarded human imagination as the essential divine quality by which God was made manifest.[5]

Neither Newton nor his *Principia* deserves the extreme judgments of Pope and Blake; the work is ranked as one of the major intellectual achievements of Western culture. The Enlightenment of the eighteenth century and the Romanticism of the nineteenth century both have their roots in the acceptance or rejection of the new worldview that paid homage to Newton's scientific writings. Just as his *Optics* provided a model for the experimental method, so his *Principia* laid the foundations for the theoretical method. It was, in fact, the wave of the future.